Federal Board 9th Class New Book

Mathematics

Notes Chapter 1

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Complex number:

The sum of real and imaginary numbers is called complex number. It is denoted by

$z = a + bi$.

Where " a " is the real part and " b " is the imaginary part of $a + bi$ For example

$z = 5 + 2i$ where real part $= 5$ and imaginary part $= 2$

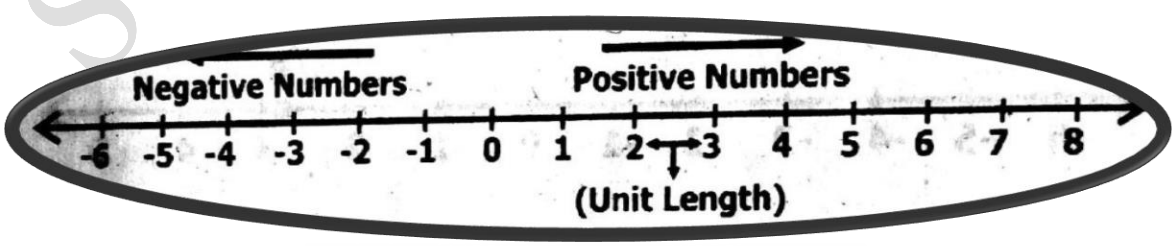
Note: The symbol " i " iota is called imaginary unit i.e. $i = (0,1)$

Its value is: $i = \sqrt{-1}$ OR $i^2 = -1$

Q. What does the term "number line" mean?

Ans: The term number line is defined as a horizontal line that displays real number including zero in ascending order.

The number line is a visualization of every real number in ascending order . Number to the left of zero are negative and right of zero is positive.

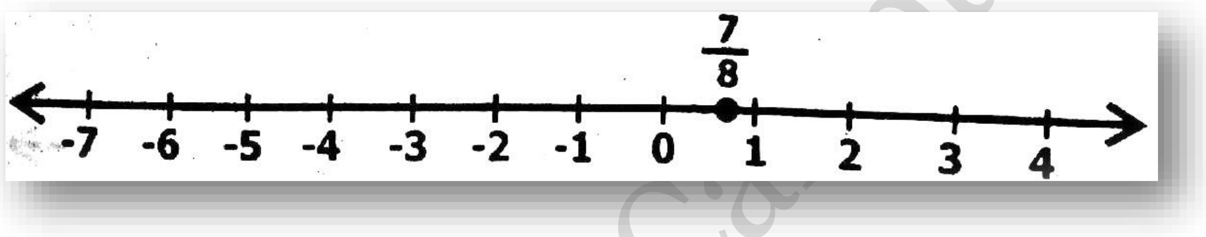


EXERCISE # 1.1

Q1. Represent each number on the number line.

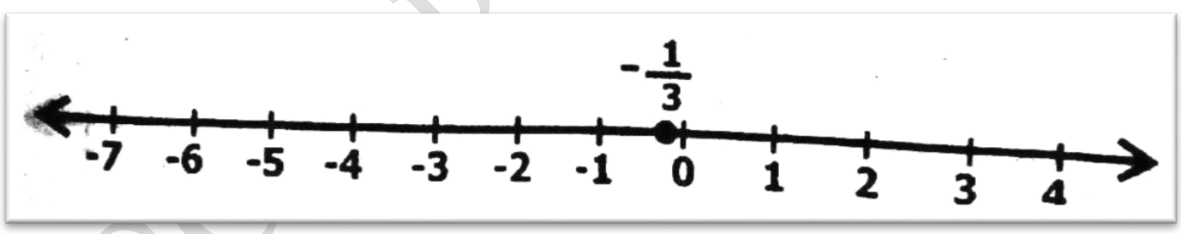
(1) $\frac{7}{8}$ =

Solution: $\frac{7}{8}$ =0.875 the point 0.8 lie between 0 and 1



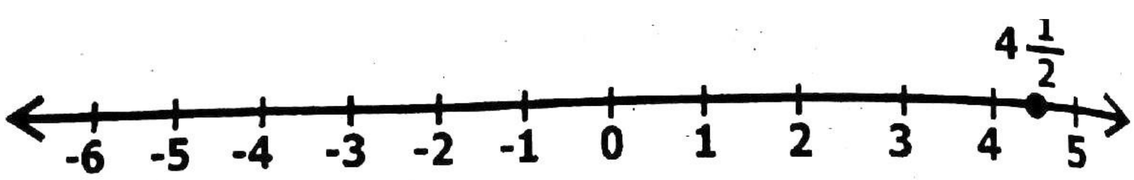
(ii) $\frac{-1}{3}$

Solution $\frac{-1}{3}$ = **-0.3333333**



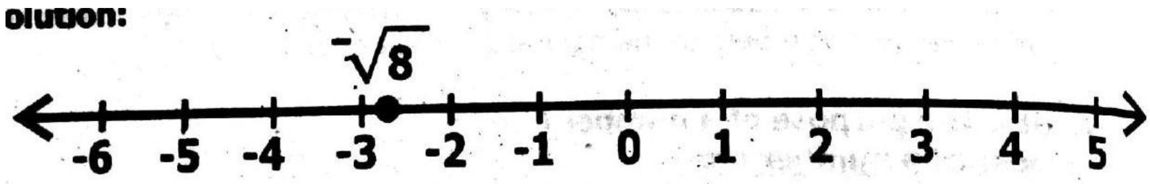
(iii) $4\frac{1}{2}$

Solution: $4\frac{1}{2}$ = $\frac{9}{2}$ = 4.5



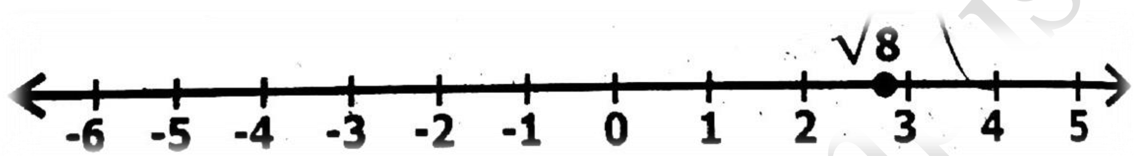
(iv) $-\sqrt{8}$

Solution: $-\sqrt{8} = -2.8$



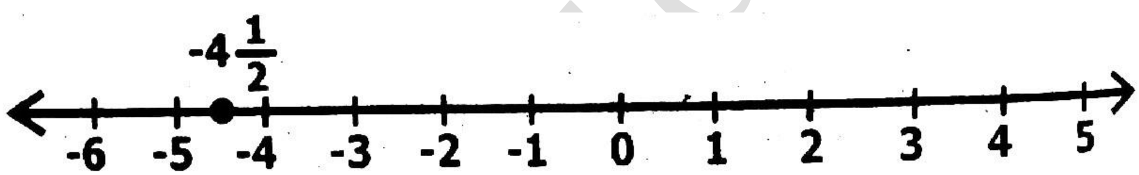
(v) $\sqrt{8}$

Solution: $\sqrt{8} = 2.8$



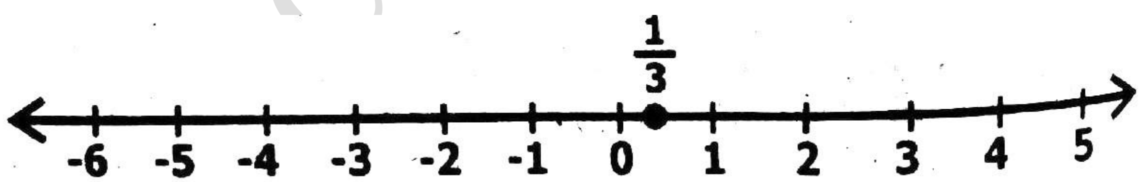
(vi) $-4\frac{1}{2}$

Solution: $-4\frac{1}{2} = -\frac{9}{2}$ = - 4.5



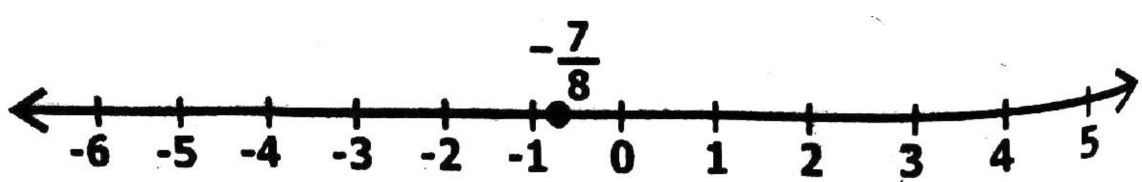
(vii) $\frac{1}{3}$

Solution: $\frac{1}{3}$ = 0.3333



(viii) $-\frac{7}{8}$

Solution: $-\frac{7}{8}$ =-0.8



PROPERTIES OF REAL NUMBERS

The basic properties of real numbers are w.r.t addition and multiplication. In tins section, some of the properties of these operations are reviewed. The following results are true for any real numbers $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{c}$.

Name of the property	With respect to		Examples	
	$+$	$\times$	$+$	$\times$
Closure	$a+b \in R$	$a \cdot b \in R$	$6+4=10 \in R$	$6 \times 4=24 \in R$
Commutative	$a+b=b+a$	$a: b=b \cdot a$	$4+7=7+4=11$	$4 \times 7=7 \times 4=28$
Associative	$a+(b+c)=$	$a \cdot(b \cdot c)=(a \cdot b) \cdot c$	$4+(6+8)=(4+6)+8=18$	$4 \times(6 \times 8)=(4 \times 6) \times 8=192$
Identity	$a+0=a=0+a$	$a \cdot 1=1 \cdot a=a$	$6+0=0+6=6$	$6 \times 1=1 \times 6=6$
Inverse	$a+(-a)=-a+a=0$	$a \cdot \frac{1}{a}=\frac{1}{a} \cdot a=1$	$14+(-14)=-14+14=0$	$14 \times \frac{1}{14}=\frac{1}{14} \times 14=1$

Q 2. Identify the property that justifies.

(i) $\mathbf{1} \times(\boldsymbol{y}-\mathbf{2})=\boldsymbol{y}-\mathbf{2}$

Solution:

Multiplicative identity

(ii) $(\mathbf{0.2}) \mathbf{5}=\mathbf{1}$

Solution:

Multiplicative inverse

(iii) $(\boldsymbol{x}+\mathbf{2})+\boldsymbol{y}=\boldsymbol{y}+(\boldsymbol{x}+\mathbf{2})$

Solution:

Commutative property w.r.t. 't'

(iv) - (3b) + (3b) = 0

Solution: Additive inverse

(v) $(\boldsymbol{x}+\mathbf{5})-\mathbf{1}=\boldsymbol{x}+(\mathbf{5}-\mathbf{1})$

Solution:

Associative property w.r.t. ' +

(vi) $-\mathbf{3}(\mathbf{2}-\boldsymbol{y})=-\mathbf{6}+\mathbf{3}\boldsymbol{y}$

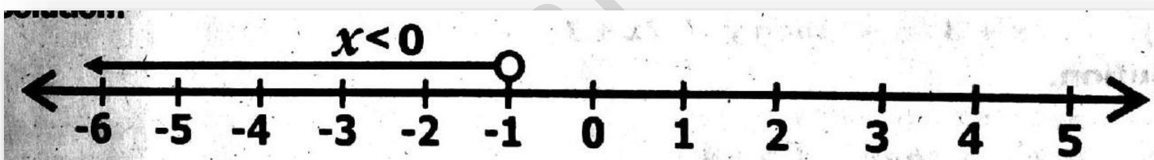
Solution:

Distributive property of multiplication over subtraction

Q 3 . Represent the following on a number line.

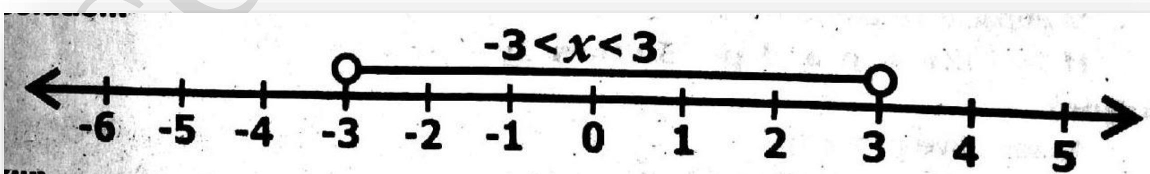
(i) $x<0$

Solution:



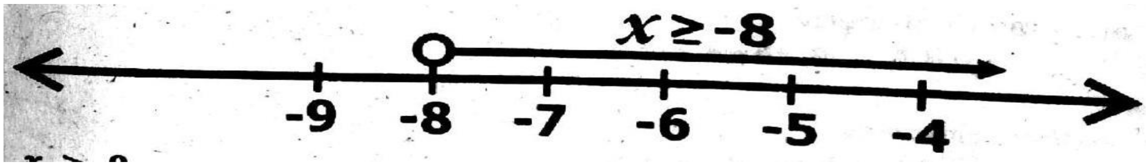
(ii) $-3<x<3$

Solution:



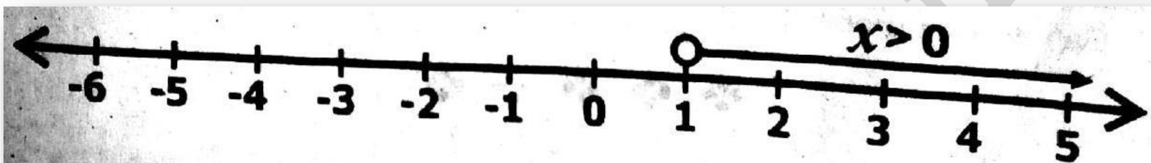
(iii) $x \geq -8$

Solution:



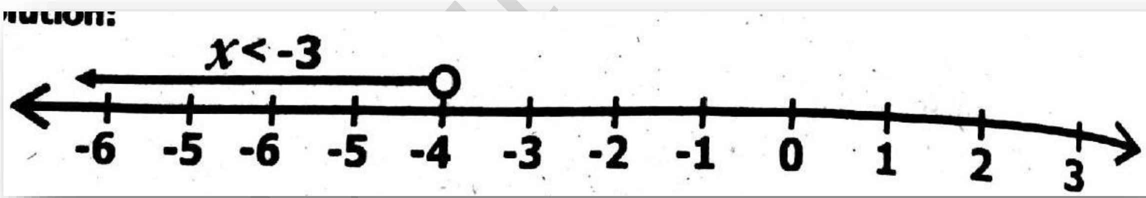
(Iv) $x > 0$

Solution:



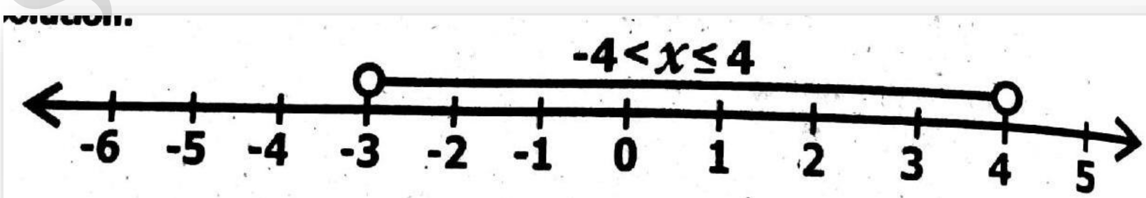
$x < -3$

Solution:



(vi) $-4 < x \leq 4$

Solution:



PROPERTIES OF INEQUALITY OF REAL NUMBERS

The following properties are true for any real numbers $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{c}$.

Name of property	General statement	Example
Trichotomy	Either $a > b$ or $a = b$ or $a < b$	If $2 < 3$, then $2 \neq 3$ and $2 > 3$
Transitive	If $a < b$ and $b < c$ then $a < c$ If $a > b$ $a > c$	If $3 < 5$ and $5 < 7$, then $3 < 7$ If $-2 > -5$ and $-5 > -7$, $2 > -7$
Additive	If $a < b$, then: $a + c < b + c$ If $a > b, b$ then: $a + c > b + c$	If $3 < 5$, then: $3 + 10 < 5 + 10$ If $-5 > -8$, then: $-5 + 2 > -8 + 2$
Multiplicative	(a) For $c < 0$ and $a, b \in R$,	

Q4. Identify the properties of equality and inequality of real numbers that justifies the statement.

(i) $9x = 9x$

Solution:

Reflexive property

(ii) If $x + 2 = y$ and $y = 2x - 3$, then $x + 2 = 2x - 3$

Solution:

Transitive property

(iii) If $2x + 3 = y$, then $y = 2x + 3$.

Solution:

Symmetric property

(iv) If $3 < 4$, then $-3 > -4$.

Solution:

Multiplicative property

(v) If $2y + 2w = p$, and $p = 50$, then $2y + 2w = 50$.

Solution:

Associative property

(vi) If $x + 4 > y + 4$, then $x > y$

Solution:

Cancellation property

(vii) If $2 < 5$ and $5 < 9$, then $2 < 9$.

Solution:

Transitive property

(vili) If $-18 < -16$, then $9 > 8$

Solution:

Cancellation property

EXERCISE # 1.2

Q1. By using the property of product and quotient rule for radicals,write each expression as a single radical and simplify.

(i) $\sqrt[3]{6} \cdot \sqrt[3]{6}$

Solution: $\sqrt[3]{6} \cdot \sqrt[3]{6}$

$$= (6)^{\frac{1}{3}} \cdot (6)^{\frac{1}{3}}$$
$$= (6)^{\frac{1}{3}+\frac{1}{3}} = 6^{\frac{1+1}{3}} = 6^{\frac{2}{3}}$$

(ii) $\sqrt[5]{4} \cdot \sqrt[5]{8}$

Solution: $\sqrt[5]{4} \cdot \sqrt[5]{8}$

$$= (4)^{\frac{1}{5}} \cdot (8)^{\frac{1}{5}}$$
$$= (2 \times 2)^{\frac{1}{5}} \cdot (2 \times 2 \times 2)^{\frac{1}{5}}$$
$$= 2^{\frac{1}{5}} \times 2^{\frac{1}{5}} \times 2^{\frac{1}{5}} \times 2^{\frac{1}{5}} \times 2^{\frac{1}{5}}$$
$$= 2^{\frac{5}{5}} = \sqrt[5]{2^5}$$

(iii) $\sqrt[4]{x} \cdot \sqrt[4]{x^3}$

Solution: $\sqrt[4]{x} \cdot \sqrt[4]{x^3}$

$$= (x^{1+3})^{\frac{1}{4}}$$

$= (x^4)^{\frac{1}{4}} = \sqrt[4]{x^4} = (x)^{4\times\frac{1}{4}}$

(iv) $\sqrt{10} \cdot \sqrt[3]{11}$

Solution: $\sqrt{10} \cdot \sqrt[3]{11}$

The Product Rule for Exponents: $\{a^m \cdot a^n = a^{m+n}\}$ is not applicable.

(v) $\frac{\sqrt[4]{x^7}}{\sqrt[4]{x^5}}$

Solution: $\frac{\sqrt[4]{x^7}}{\sqrt[4]{x^5}}$

$$= \frac{(x^7)^{\frac{1}{4}}}{(x^5)^{\frac{1}{4}}}$$

$(x^{7-5})^{\frac{1}{4}} = (x^2)^{\frac{1}{4}} = \sqrt[4]{x^2}$ or $x^{2\times\frac{1}{4}} = x^{\frac{1}{2}}$

(vi) $\frac{\sqrt[3]{5000}}{\sqrt[3]{5}}$

Solution: $\frac{\sqrt[3]{5000}}{\sqrt[3]{5}}$

$$= \frac{(5 \times 10^3)^{\frac{1}{3}}}{(5)^{\frac{1}{3}}}$$

$$\frac{(5)^{\frac{1}{3}} \cdot (10^3)^{\frac{1}{3}}}{(5)^{\frac{1}{3}}} = (5)^{\frac{1}{3}-\frac{1}{3}} \cdot (10)^1$$

$$5^0 \times 10^1 = 1 \times 10 = 10$$

or

Also; 10 can be written as: $\sqrt[3]{10^3}$

(vii) $\frac{\sqrt[2]{500}}{\sqrt[2]{5}}$

Solution: $\frac{\sqrt[2]{500}}{\sqrt[2]{5}}$

$$= \frac{(5 \times 10^2)^{\frac{1}{2}}}{(5)^{\frac{1}{3}}} = \frac{5^{\frac{1}{2}} \times (10^2)^{\frac{1}{2}}}{5^{\frac{1}{2}}}$$

$$5^{\frac{1}{2}-\frac{1}{2}} \times 10^1 = 5^0 \times 10^1 = 1 \times 10 = 10$$

or Also; 10 can be written as: $\sqrt[2]{10^2}$

(viii) $\sqrt[2]{10} \cdot \sqrt[3]{7}$

Solution: $\sqrt[2]{10} \cdot \sqrt[3]{7}$

The Product Rule for Exponents: $\{a^m \cdot a^n = a^{m+n}\}$ is not applicable.

Q2. Write each exponential expression as an equivalent radial expression and simplify If possible.

(1) $(216)^{\frac{2}{3}}$

Solution: $(216)^{\frac{2}{3}}$

$$\text{Radical Expression } = (216)^{2 \times \frac{1}{3}}$$

$$= \sqrt[3]{(216)^2}$$

$$= (6 \times 6 \times 6)^{\frac{2}{3}} = (6^3)^{\frac{2}{3}}$$

$$= 6^2 = 36$$

(ii) $(29)^{\frac{1}{2}}$

Solution: $(29)^{\frac{1}{2}}$

Radical Expression $\sqrt{29}$ not simplifying

(iii) $\left(\frac{1}{32}\right)^{\frac{1}{5}}$

Solution:

$$\left(\frac{1}{32}\right)^{\frac{1}{5}}$$

$$\text{Radical Expression } = \sqrt[5]{\frac{1}{32}}$$

$$= \left(\frac{1}{32}\right)^{\frac{1}{5}} = \frac{(1)^{\frac{1}{5}}}{(2 \times 2 \times 2 \times 2 \times 2)^{\frac{1}{5}}}$$

$$= \frac{1}{(2^5)^{\frac{1}{5}}} = \frac{1}{2}$$

(iv) $(\mathbf{216})^{\frac{-2}{3}}$

Solution: $(216)^{\frac{-2}{3}}$

$$\text{Radical Expression } = (216)^{-2\times\frac{1}{3}} = \sqrt[3]{(216)^{-2}}$$

$$= (216)^{-\frac{2}{3}} = \frac{1}{(216)^{\frac{2}{3}}} = \frac{1}{(6^3)^{\frac{2}{3}}}$$

$$= \frac{1}{6^2} = \frac{1}{36}$$

(v) $(\mathbf{1000})^{\frac{1}{3}}$

Solution: $(1000)^{\frac{1}{3}}$

$$\text{Radical Expression } = \sqrt[3]{1000}$$

$$= (10 \times 10 \times 10)^{\frac{1}{3}} = (10^3)^{\frac{1}{3}} = 10$$

(vi) $\left(\frac{1}{39}\right)^{-\frac{1}{2}}$

Solution: $\left(\frac{1}{39}\right)^{-\frac{1}{2}} = (39)^{\frac{1}{2}} = \sqrt[2]{39}$ or $\sqrt{39}$

Q3. Write Radical Expression into Exponential Expression:

(1) $(\sqrt[3]{\mathbf{5}})^{\mathbf{2}}$

Solution: $(\sqrt[3]{5})^2$

Exponential Expression:

$\left(5^{\frac{1}{3}}\right)^2 = 5^{\frac{2}{3}}$

(ii) $(\sqrt[4]{\mathbf{10}})^{\mathbf{8}}$

Solution: $(\sqrt[4]{10})^8 = \left(10^{\frac{1}{4}}\right)^8$

Exponential Expression:

$$10^2 = 10 \times 10 = 100$$

(iii) $-(\sqrt[3]{\mathbf{6}})^{\mathbf{6}}$

Solution: $-(\sqrt[3]{6})^6 = -\left(6^{\frac{1}{3}}\right)^6 = -(6)^2$

Exponential Expression:

$$-6^2 = -(6 \times 6) = -36$$

(iv) $(\sqrt[3]{\mathbf{6}})^{\mathbf{6}}$

Solution: $(\sqrt[3]{6})^6 = \left(6^{\frac{1}{3}}\right)^6 = 6^2$, $6^2 = 6 \times 6 = 36$

(v) $-(\sqrt[3]{\mathbf{5}})^{\mathbf{2}}$

Solution: $-(\sqrt[3]{5})^2 = -\left(5^{\frac{1}{3}}\right)^2 = -5^{\frac{2}{3}}$

(vi) $-(\sqrt[4]{\mathbf{10}})^{\mathbf{8}}$

Solution: $-(\sqrt[4]{10})^8 = \left(10^{\frac{1}{4}}\right)^8 = -(10^2) = -10^2 = -(10 \times 10) = -100$

Q4. Use the properties of exponents to simplify each of the following Assume that all variables represent positive numbers. (Write all results with positive exponents.)

(i) $\dfrac{(16)^{\frac{1}{5}}\cdot(16)^{\frac{1}{4}}}{(16)^{\frac{-3}{10}}}$

Solution: $\dfrac{(16)^{\frac{1}{5}}\cdot(16)^{\frac{1}{4}}}{(16)^{\frac{-3}{10}}}$

$$= 16^{\frac{1}{5}+\frac{1}{4}+\frac{3}{10}} = 16^{\frac{8+10+12}{40}}$$
$$= 16^{\frac{30}{40}} = 16^{\frac{3}{4}} = (2^4)^{\frac{3}{4}} = 2^3$$

(ii) $7^{\frac{-1}{3}}\left(7^{\frac{5}{3}} - 7^{\frac{4}{3}}\right)$

Solution: $7^{\frac{-1}{3}}\left(7^{\frac{5}{3}} - 7^{\frac{4}{3}}\right)$

$= 7^{-\frac{1}{3}}\cdot 7^{\frac{5}{3}} - 7^{\frac{4}{3}}\cdot 7^{-\frac{1}{3}}$

$= 7^{\frac{5}{3}-\frac{1}{3}} - 7^{\frac{4}{3}-\frac{1}{3}} = 7^{\frac{5-1}{3}} - 7^{\frac{4-1}{3}}$

$= 7^{\frac{4}{3}} - 7^{\frac{3}{3}} = 7^{\frac{4}{3}} - 7^1$

$= 7^{\frac{4}{3}} - 7$

(iii) $\dfrac{2^{\frac{2}{3}}\cdot 2^{\frac{1}{7}}}{2^{\frac{1}{2}}}$

Solution: $\dfrac{2^{\frac{2}{3}}\cdot 2^{\frac{1}{7}}}{2^{\frac{1}{2}}}$

$$= 2^{\frac{2}{3}+\frac{1}{7}-\frac{1}{2}} = 2^{\frac{28+6-21}{42}} = 2^{\frac{34-21}{42}} = 2^{\frac{13}{42}}$$

(iv) : $\dfrac{3^{\frac{-1}{2}}\cdot 3^{\frac{1}{2}}}{3^{\frac{1}{2}}}$

Solution: $\dfrac{3^{\frac{-1}{2}}\cdot 3^{\frac{1}{2}}}{3^{\frac{1}{2}}}$

$$= 3^{-\frac{1}{2}+\frac{1}{2}-\frac{1}{2}} = 3^{-\frac{1}{2}}$$
$$= \frac{1}{3^{\frac{1}{2}}} = \frac{1}{\sqrt{3}}$$

(v) $\left(\frac{36^{\frac{1}{2}}\cdot 6^{\frac{1}{2}}}{8^{\frac{1}{2}}\cdot 27^{\frac{1}{2}}}\right)^{3}$

Solution: $\left(\frac{36^{\frac{1}{2}}\cdot 6^{\frac{1}{2}}}{8^{\frac{1}{2}}\cdot 27^{\frac{1}{2}}}\right)^{3}$

$$=\left(\frac{(6^2)^{\frac{1}{2}}\cdot 6^{\frac{1}{2}}}{(2^3)^{\frac{1}{2}}\cdot (3^3)^{\frac{1}{2}}}\right)^{3}=\left(\frac{6\cdot 6^{\frac{1}{2}}}{2^{\frac{3}{2}}\cdot 3^{\frac{3}{2}}}\right)^{3}=\frac{(6^2)^{\frac{3}{2}}\cdot 6^{\frac{3}{2}}}{2^{3\times\frac{3}{2}}\cdot (3^3)^{\frac{3}{2}}}$$

$$=\frac{6^3\cdot 6^{\frac{3}{2}}}{2^{\frac{9}{2}}\cdot 3^{\frac{9}{2}}}=\frac{(2\times 3)^3\cdot (2\times 3)^{\frac{3}{2}}}{2^{\frac{9}{2}}\cdot 3^{\frac{9}{2}}}$$

$$=\frac{2^3\cdot 3^3\cdot 2^{\frac{3}{2}}\cdot 3^{\frac{3}{2}}}{2^{\frac{9}{2}}\cdot 3^{\frac{9}{2}}}=2^{3+\frac{3}{2}-\frac{9}{2}}\cdot 3^{3+\frac{3}{2}-\frac{9}{2}}$$

$$=2^{\frac{6+3-9}{2}}\times 3^{\frac{6+3-9}{2}}=2^{\frac{9-9}{2}}\times 3^{\frac{9-9}{2}}$$

$$=2^0\times 3^0=1\times 1=1$$

(vi) $\left(\frac{2187a^5b^{17}}{a^{12}b^3}\right)^{\frac{1}{7}}$

Solution: $\left(\frac{2187a^5b^{17}}{a^{12}b^3}\right)^{\frac{1}{7}}$

$$=\left(\frac{3^7a^5b^{17}}{a^{12}b^3}\right)^{\frac{1}{7}}=\left(\frac{3^7\cdot b^{17-3}}{a^{12-5}}\right)^{\frac{1}{7}}=\left(\frac{3^7\cdot b^{14}}{a^7}\right)^{\frac{1}{7}}$$

$$=\frac{(3^7)^{\frac{1}{7}}\cdot (b^{14})^{\frac{1}{7}}}{(a^7)^{\frac{1}{7}}}=\frac{3b^2}{a}$$

(vii) $\sqrt[4]{\frac{a^3}{b^3}}\times\sqrt[4]{\frac{b^3}{c^3}}\times\sqrt[4]{\frac{c^3}{a^3}}$

Solution: $\sqrt[4]{\frac{a^3}{b^3}}\times\sqrt[4]{\frac{b^3}{c^3}}\times\sqrt[4]{\frac{c^3}{a^3}}=\left(\frac{a^3}{b^3}\right)^{\frac{1}{4}}\cdot\left(\frac{b^3}{c^3}\right)^{\frac{1}{4}}\cdot\left(\frac{c^3}{a^3}\right)^{\frac{1}{4}}$

$$=\left(\frac{a^3}{b^3}\cdot\frac{b^3}{c^3}\cdot\frac{c^3}{a^2}\right)^{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}=(a^{3-3}\cdot b^{3-3}\cdot c^{3-3})^{\frac{3}{4}}$$

$$=(a^0\cdot b^0\cdot c^0)^{\frac{3}{4}}=(1\times 1\times 1)^{\frac{3}{4}}$$

$$=(1)^{\frac{3}{4}}=1$$

Q5. Use suitable laws of exponents to show that

$$\left(\frac{x^p}{x^q}\right)^{p+q} \times \left(\frac{y^q}{y^r}\right)^{q+r} \times \left(\frac{z^r}{z^p}\right)^{r+p} \times x^{q^2} \times y^{r^2} \times z^{p^2} = x^{p^2} \times y^{q^2} \times z^{r^2}$$

Solution: L.H.S. $= \left(\frac{x^p}{x^q}\right)^{p+q} \times \left(\frac{y^q}{y^r}\right)^{q+r} \times \left(\frac{z^r}{z^p}\right)^{r+p} \times x^{q^2} \times y^{r^2} \times z^{p^2}$

$= (x^p \cdot x^{-q})^{p+q} \times (y^q \cdot y^{-r})^{q-r} \times (z^r \cdot z^{-p})^{r+p} \times x^{q^2} \times y^{r^2} \times z^{p^2}$

$= (x^{p-q})^{p+q} \times (y^{q-r})^{q+r} \times (z^{r-p})^{r+p} \times x^{q^2} \times y^{r^2} \times z^{p^2}$

$= x^{p^2-q^2} \times y^{q^2-r^2} \times z^{r^2-p^2} \times x^{q^2} \times y^{r^2} \times z^{p^2}$

$= x^{p^2-q^2} \cdot x^{q^2} \times y^{q^2} - r^2 \cdot y^{r^2} \times z^{r^2-p^2} \cdot z^{p^2}$

$= x^{p^2-q^2+q^2} \times y^{q^2-r^2-r^2} \times z^{r^2-p^2-p^2}$

$= x^{p^2} \times y^{q^2} \times z^{r^2}$ = R.H.S

∴ L.H.S. = R.H.S.

EXERCISE # 1.3

1. On his last bank statement, Qasim had a balance of Rs. $1, 75, 000$ in his checking account. He wrote one cheque for Rs. 45790 and another for Rs. 112,921. What is his current balance?

Solution:

Initial balance = Rs. 1, 75, 000

Cheque 1 = Rs. 45,790

Cheque 2 = Rs. 112, 921

Total amount of cheques = Rs. 45,790 + Rs. 112,921 = Rs. 158, 711

Now, subtract the total amount of the cheques from the initial balance:

Current balance = Rs. 1, 75, 000 - Rs. 158,711 = Rs. 16, 289

Therefore, Qasim's current balance is Rs. 16, 289.

Q2. Last week Wajid drove 283.4 km on 16.2 litres of petrol. He says that he averaged about 1.75 km /liter. Is his answer reasonable? Explain.

Solution:

Wajid drove 283.4 km on 16.2 litres of petrol. To find the average kilometers per.liter, we divide the total kilometers driven by the total liters of petrol used:

Average fuel efficiency $= \frac{\text{Total kilometers driven}}{\text{Total litres of petrol used}}$

Average fuel efficiency $= \frac{283.4 \text{ km}}{16.2 \text{ litres}}$

Average fuel efficiency $= 17.5$ km/ litre

Wajid's actual average fuel efficiency was 17.5 km / litre, not 1.75 km /Litre

Q3.Salma bought 3.2 yard of fabric for a total price of Rs. 139.2 How much did the fabric cost per yard?

Solution:

Total price = Rs. 139.2

Total yards $= \mathbf{3.2}$

Cost per yard $= \frac{139.2}{3.2} =$ Rs. 43.5

Therefore, the fabric cost Rs. 43.5 per yard.

Q4. Momina walks . 5 km/h. She took a 12 h walk. How far did she walk .

Solution:

Walking speed = 3.5 km/h.

Time spent walking = 12 hours

Distance walked = Speed × Time = 3.5 km/h × 12 h = 42kmn Therefore, Momina walked 42 km.

Q5. The hiking club went on a 7day trip. Each day they hiked between 5.5 and 7.5 miles. It is reasonable to assume that clubbing the days the club hiked.
a. Less man 35 miles b. Between 35 and 55 miles

c. Equally 55 miles d. More than 55 miles

Solution:

Minimum distance (if they hiked 5.5 miles every day):

= 5.5 miles / day × 7 days = 38.5 miles

Maximum distance (if they hiked 7.5 miles every day):

= 7.5 miles / day × 7 days = 52.5 miles

a. Less than 35 miles:

This is not reasonable because the minimum, possible distance (38.5 miles) is already more than 35 miles.

b. Between 35 and 55 miles:

This is reasonable since both the minimum (38.5 miles) and the maximum (52.5 miles) possible distances fall within this range.

c. Exactly 55 miles:

This is not reasonable since the maximum distance they could have hiked based on the given range, is $\mathbf{52.5}$ miles.

d. More than 55 miles

This is not reasonable since the maximum distance they could have hiked based on the given range, is $\mathbf{52.5}$ miles. So no more then 55 miles.

Q6. For a class party the student's council purchased 42 balloons at Rs. 1.85 each. What is the total amount the student council paid for the balloons?

Solution ;

Purchased balloons = 42

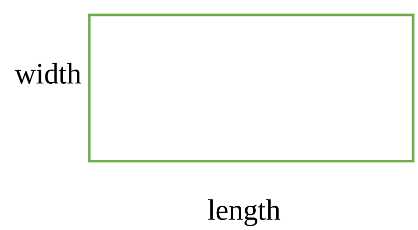
Cost of 1 balloon = 1.85

Total amount the student council paid for the balloons = 42 * 1.85 = 77.70

Q7. A group of friends made 4 yard long rectangular banner. They paid Rs.3.75 per yard for the fabric and Rs.9 for the firm to go around the banner, 10 yard perimeter. What was the width of the banner?

Given:

- Length $(l) = 4$ yards
- Perimeter $(P) = 10$-yards
- Cost of fabric $=$ Rs. 3.75 per yard



The formula for the perimeter of a rectangle is given by:

$P = 2(l + w)$

The forming the given values into the formula we get:

$10 = 2(4 + W)$

$10 = 8 + 2W$

$10 - 8 = 2W$

$2 = 2W$

$W = 1$

Therefore, the width of the banner is 1 yard.

Q8. A shoe factory has an asset for Rs. 2000,000 of which $\frac{3}{5}$ is the 'capital and rest is the debt. Find the amount of capital and debt. (Asset = capital + debt)

Solution:

Total assets $=$ Rs. $2{,}000{,}000$

Fraction of assets that is capital $= \frac{3}{5}$

First, let's calculate the amount of capital:

Capital $=$ Total assets $\times \frac{3}{5}$

Capital $= 2{,}000{,}000 \times \frac{3}{5}$

Capital = Rs. $\mathbf{1,200,000}$

Debt = total assets – capital

Debt = Rs. $\mathbf{2,000,000}$ - Rs. $\mathbf{1,200,000}$ = 800000

Q9. World lowest temperature in past 100 years was recorded to be -89.2 ^{0}C at Vostok, Antarctica on july 21, 1983. Convert this Temperature into Fahrenheit and Kelvin scales.

Ans ; Converting Temperature into Fahrenheit = Fahrenheit $({}^\circ F) = \frac{9}{5}\mathrm{C} + 32$

$$\text{Fahrenheit } ({}^\circ F) = \left(\frac{9}{5}(-89.2)\right) + 32 = -128.32{}^\circ F$$

Converting Temperature into Kelvin scales = ^{0}C + 273 = -89.2 + 273=183.8K

Q10. A company was penalized by the government act for law quality production. If the company has 3 share holders. Farah, Maryam and Tehreem investing in the ratio of 1:2:3 and the amount of penalty is Rs 456868.97. Find the amount of penalty paid by each of 3 share holders.

Ans; Total amount of penalty = Rs 456868.97

Investing ratio of 3 share holder = 1:2:3 = Farah:Maryam:Tehreem

Sum of Ratio = 1+2+3 = 6

Amount Paid by Farah = $\frac{1}{6} \times 456868.97 = 76144.83$

Amount Paid by Farah = $\frac{2}{6} \times 456868.97 = 152289.66$

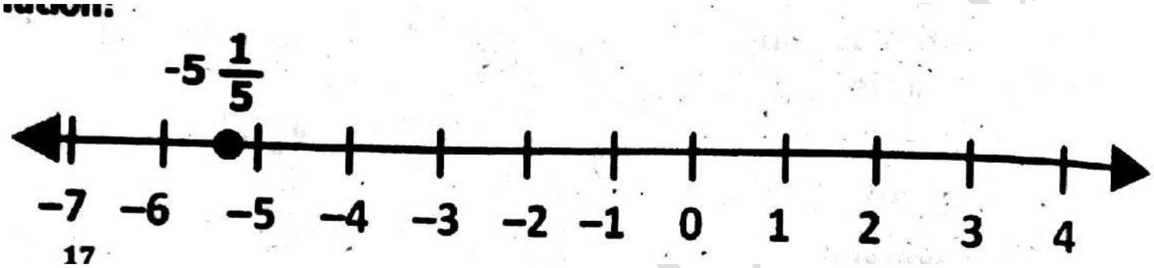
Amount Paid by Farah = $\frac{3}{6} \times 456868.97 = 228434.49$

Review **EXERCISE**

Q2. Represent each number on the number line.

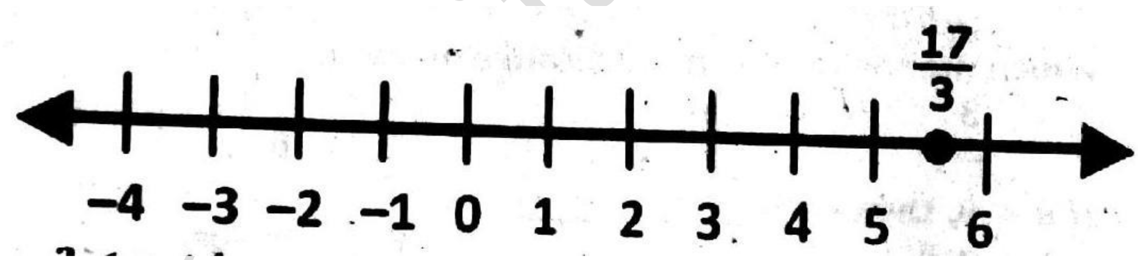
Solution:

(i) $-5\frac{1}{5}$



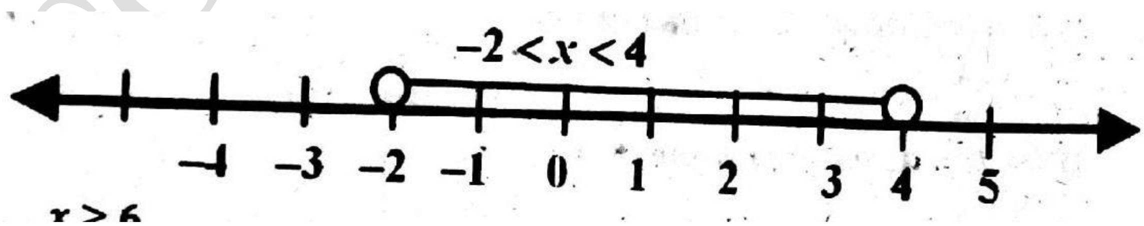
(ii) $\frac{17}{3}$

Solution:



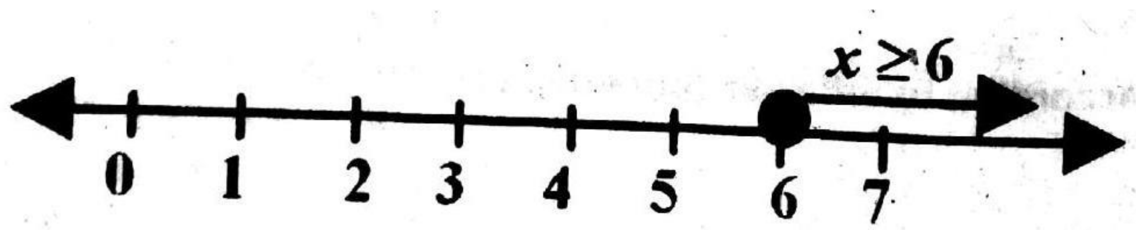
(iii) $-2 < x < 4$

Solution:



(Iv) $x \geq 6$

Solution:



Q3. Write each exponential expression as an equivalent radical expression and simplify if possible.

(i) $(-\mathbf{2})^{\frac{4}{5}}$

Solution: $(-2)^{\frac{4}{5}}$

$$(-2)^{4\times\frac{1}{5}} = [(-2)(-2)(-2)(-2)]^{\frac{1}{5}}$$
$$= (16)^{\frac{1}{2}} = \sqrt[5]{16}$$

(ii) $(-\mathbf{2},\mathbf{7})^{\frac{1}{3}}$

solution: $(-27)^{\frac{1}{3}}$

$$(-27)^{3}$$
$$= [(-3)(-3)(-3)]^{\frac{1}{3}} = [(-3)^{3}]^{\frac{1}{3}} = \sqrt[3]{(-3)^{3}}$$

(iii) $(\sqrt{\mathbf{16}})^{\mathbf{4}}$

solution: $(\sqrt{16})$

$$= \sqrt{(2\times 2\times 2\times 2)^{4}}$$
$$= \sqrt{(2^{4})^{4}} = \sqrt{(2)^{16}} = (2)^{16\times\frac{1}{2}}$$
$$= 2^{8} = 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2 = 128$$

(iv) $(\sqrt[3]{-\mathbf{8}})^{\mathbf{9}}$

Solution: $(\sqrt[3]{-8})^{9}$

$$= (\sqrt[3]{(-2)(-2)(-2)})^{9}$$
$$= [(-2)(-2)(-2)]^{9\times\frac{1}{3}}$$
$$= (-2)^{3\times 9\times\frac{1}{3}} = (-2)^{9}$$
$$= (-2)(-2)(-2)(-2)(-2)(-2)(-2)(-2)(-2)$$
$$= -256$$

(v) $\left(x^{-2}\right)^{3}\cdot\left(x^{0}\right)^{5}$

Solution: $(x^{-2})^3 \cdot (x^0)^5$

$$= (x^{-6}) \cdot (1)^5$$
$$(x^{-6})(1) = x^{-6} = \frac{1}{x^6}$$

Q4. Ulse the properties of exponents to simplify each of the following.

(i) $\frac{(-2)^3 \cdot (-2)^{-4} \cdot (-2)}{(-2)^{-3}}$

Solution: $\frac{(-2)^3 \cdot (-2)^{-4} \cdot (-2)}{(-2)^{-3}}$

$= (-2)^{3-4+1+3}$

$= (-2)^{7-4} = (-2)^3$

$= (-2)(-2)(-2) = -8$

(ii) $\frac{2^{\frac{1}{2}} \cdot 2^{\frac{3}{4}}}{2^{\frac{1}{2}}} \times \frac{3.3^{\frac{3}{2}}}{3^{\frac{1}{2}}}$

Solution: $\frac{2^{\frac{1}{2}} \cdot 3^{\frac{3}{4}}}{2^{\frac{1}{2}}} \times \frac{3.3^{\frac{3}{2}}}{3^{-\frac{1}{2}}}$

$= 2^{\frac{1}{2}+\frac{3}{4}-\frac{1}{2}} \times 3^{1+\frac{3}{2}+\frac{1}{2}}$

$$= 2^{\frac{2+3-2}{4}} \times 3^{\frac{2+3+1}{2}} = 2^{\frac{3}{4}} \times 3^{\frac{6}{2}}$$
$$= 2^{\frac{3}{4}} \times 3^3$$

Q5. Determine whether each statement is true or false. If false, give example of a number that shows the statement is true.

A. Every rational number is an integer. ⇒ (False)

Not every rational number is an integer. Rational numbers include integer but they also include fractions. For example, $\frac{3}{2}$ is a rational number but n an integer.

B. Every real number is an irrational number. ⇒ (False)

Real numbers include both rational and irrational numbers. An irration number cannot. be expressed as a fraction of two integers. However, rationa numbers can be expressed as fractions of two integers. For example, $\sqrt{2}$ an irrational number, but $\frac{3}{2}$ is a rational number.

C. Every irrational number is a real number. ⇒ (True)

Irrational numbers are real numbers that cannot be expressed as fractions two integers. All real numbers include both rational and irrational numbers.

D. ' Every integer is a rational number. ⇒ (True)

Every integer can be expressed as a fraction where the denominator is 1 For example, $3 = \frac{3}{1}, -5 = \frac{-5}{1}$, etc.

E. Every real number is either a rational number or an irrationd number. ⇒ (True)

Every real number can be classified as either rational or irrational. Ration numbers are those that can be expressed as fractions of two integers, write irrational numbers cannot be expressed as such. Since every real numb falls into one of these categories, this statement is true.